

VICTORIA JUNIOR COLLEGE
Preliminary Examination

MATHEMATICS

Higher 1

Paper 1

8863/01

September 2007

3 hours

Additional materials: Answer paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages and **1** blank page.

Victoria Junior College

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[Turn over

Section A: Pure Mathematics [40 marks]

- 1 On a single diagram sketch the graph of $y = |x + 1|$ and $y = e^{1-|x|}$. [2]

Hence solve the inequality

$$|x + 1| \leq e^{1-|x|}. \quad [3]$$

- 2 A function f is defined by

$$f: x \mapsto x^2 + ax, \quad x \in [b, \infty),$$

where a is a positive constant.

- (i) Given that f^{-1} exists, find, in terms of a , the least value of b . [3]
- (ii) Given that $(3, 1)$ is a point on the graph of $y = f^{-1}(x)$, find the value of a . [3]

3

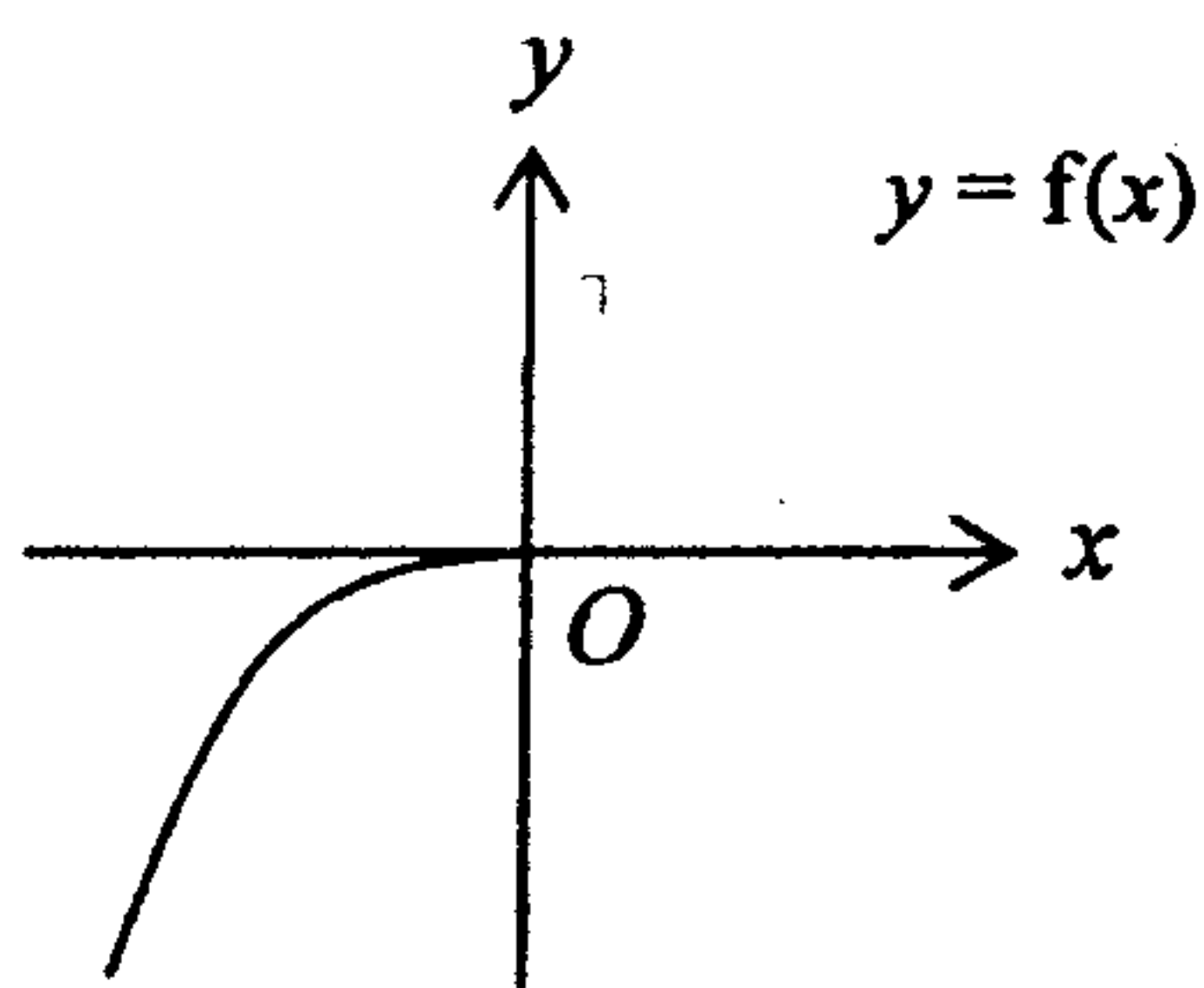


Diagram 1

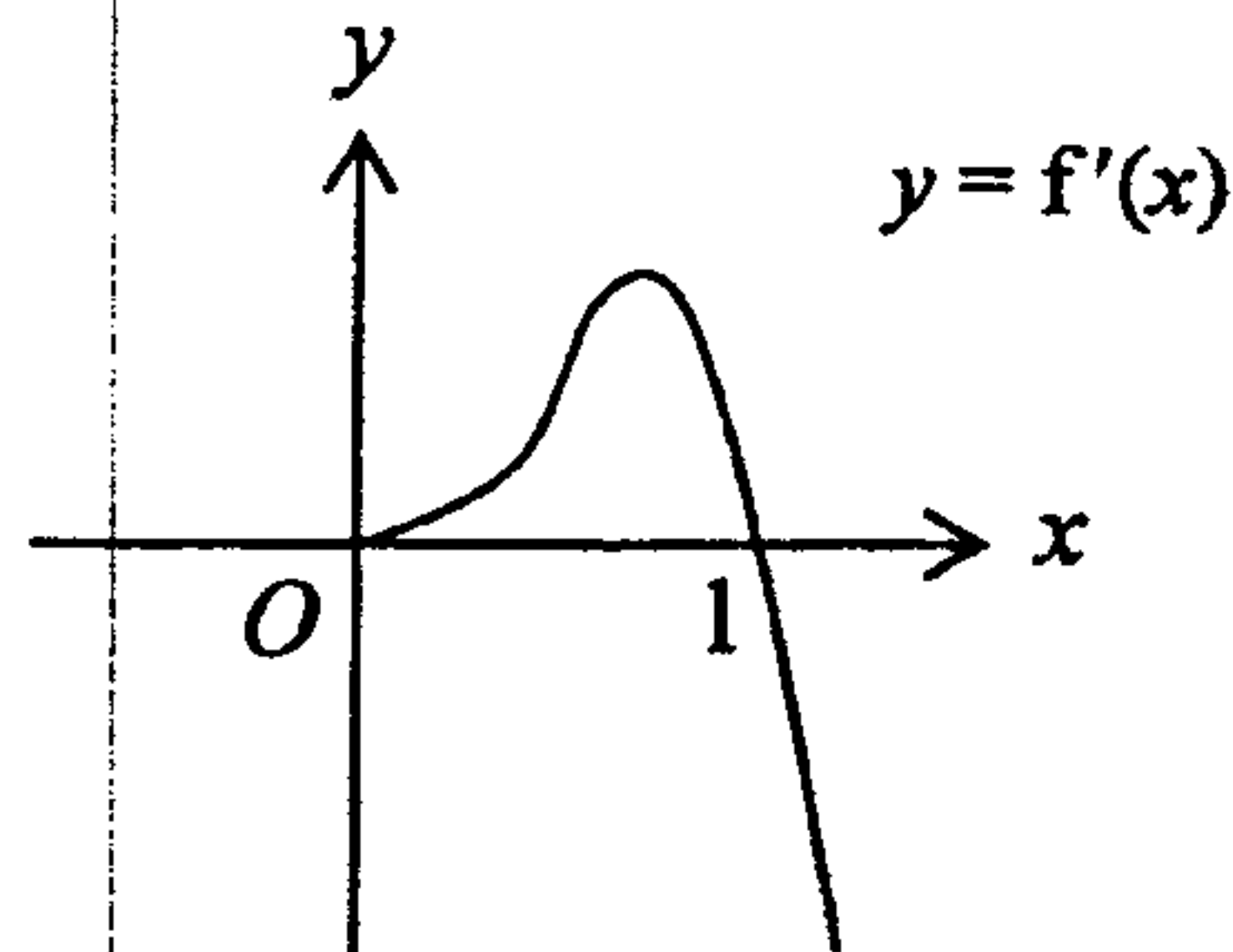


Diagram 2

Diagram 1 shows part of the graph of $y = f(x)$ and diagram 2 shows part of the graph of $y = f'(x)$. Both graphs pass through the origin and $f(1) = 1$ and $f'(1) = 0$.

Copy and complete both graphs. [3]

Shade on an appropriate graph, the area represented by the integral

$$\int_0^1 f'(x) \, dx. \quad [2]$$

Hence, find the value of this area. [2]

4 An odd-shaped device is transforming constantly, and at any time t seconds, its surface area is given by $A = \ln(3t+1) \text{ cm}^2$ and its volume is given by $V = e^{0.2t} - 1 \text{ cm}^3$.

(i) Show that both its surface area and its volume are strictly increasing with time. [5]

(ii) Find the rate of increase of its volume at the instant when the rate of increase of its surface area is $0.1875 \text{ cm}^2 \text{ s}^{-1}$. [3]

5 The equation of a curve is given by $y = \frac{2-x}{x^2}$.

(i) Use a non-calculator method to find the coordinates of the minimum point on the curve. [3]

(ii) Find the equation of the normal to the curve at $x = 3$. [4]

(iii) Find the exact value of the area of the region enclosed by the curve, the x -axis and the lines $x = 1$ and $x = 2$. [4]

Consider the equation $\frac{2-x}{x^2} = mx$, where m is an integer. Find all possible values of m if this equation has a root between 0.6 and 0.7. [3]

[Turn over

Section B: Statistics [60 marks]

6 A biased coin is flipped n times and the number of heads obtained is denoted by X . If $E(X) = 12$ and $\text{Var}(X) = 4$, find the value of n . [4]

7 The mass of the students in a certain school has a mean of 51 kg and a variance of 121 kg^2 . A school bus with a maximum safety load of 2600 kg is used to transport 50 randomly selected students from this school. Find the probability that the maximum safety load will be exceeded. [5]

8 In a small town, the following information is available from the registry office:

Race	Number of residents
Chinese	850
Malay	75
Indian	50
Eurasian	25
Total	1000

10 residents are chosen from each race for an interview.

- (i) Identify the type of sampling used. [1]
- (ii) Determine, with a reason, whether this sampling method is random. [2]
- (iii) Explain in detail how the sampling method can be improved. [3]

9 The mass, x g, of the content of each packet of potato chips in a random sample of 50 packets is measured, and the result is summarised by

$$\sum(x - 200) = -25, \quad \sum(x - 200)^2 = 159.5.$$

Test, at the 3 % significance level, whether the population mean mass of the content is less than 200 g. [4]

The same sample is used in another test to determine whether the mean mass of the content is 200g. If the alternative hypothesis is accepted, determine an inequality satisfied by the significance level, α %, of this test. [2]

10 The mass of a randomly chosen pear of variety A has a normal distribution with mean 115 g and standard deviation 26 g. The mass of a randomly chosen pear of variety B has a normal distribution with mean 104 g and standard deviation 21 g. Find the probability that the mean mass of 3 randomly chosen pears of variety A is greater than the mean mass of 4 randomly chosen pears of variety B . [7]

11 A bag contains 10 red balls, 5 green balls and 3 blue balls. 2 balls, one after another, are selected at random without replacement.

- (i) Find the probability that the 2 balls selected are of the same colour. [3]
- (ii) Given that the first ball selected is red, find the probability that the second ball selected is not red. [3]
- (iii) Given that the second ball selected is red, find the probability that the first ball selected is red. [3]

12 X follows a normal distribution with mean μ and variance σ^2 . If $P(X > 10) = 0.179$ and $P(X < 9) = 0.720$, find, correct to 3 significant figures, the values of μ and σ^2 . [4]

6 observations of X are taken. Find the probability that at least 2 of the observations will be more than 10. [3]

60 observations of X are taken. Using a suitable approximation, find the probability that at least 44 of the observations will be less than 9. [4]

13 In an attempt to investigate the relationship between the length of time spent on revision for a Mathematics examination and the actual mark obtained at the examination, a set of data was collected from 8 randomly selected students. The time spent, x (in weeks), and the mark obtained, y , is given below:

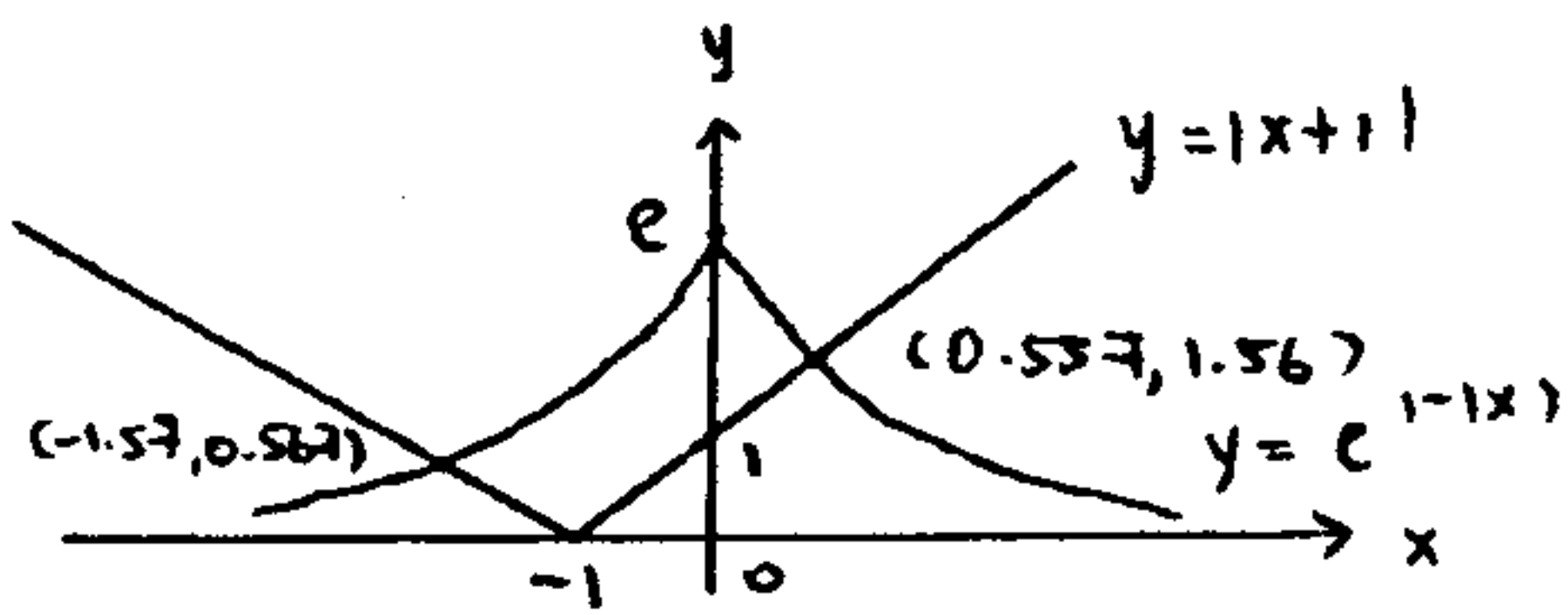
x	6	7	5	2	9	8	4	3
y	50	62	48	18	78	72	73	25

- (i) Calculate the value of r , the linear (product moment) correlation coefficient for this set of data. [2]
- (ii) Give a sketch of the scatter diagram for the data. Giving a reason, identify a data pair which should be regarded as suspect. [3]
- (iii) Remove the suspect data pair and calculate the new correlation coefficient. Comment on its value. [2]
- (iv) Based on this new set of data, estimate the mark obtained by a student who spent
 - (a) 4 weeks on the revision,
 - (b) 12 weeks on the revision.

Comment on the reliability of your answers. [5]

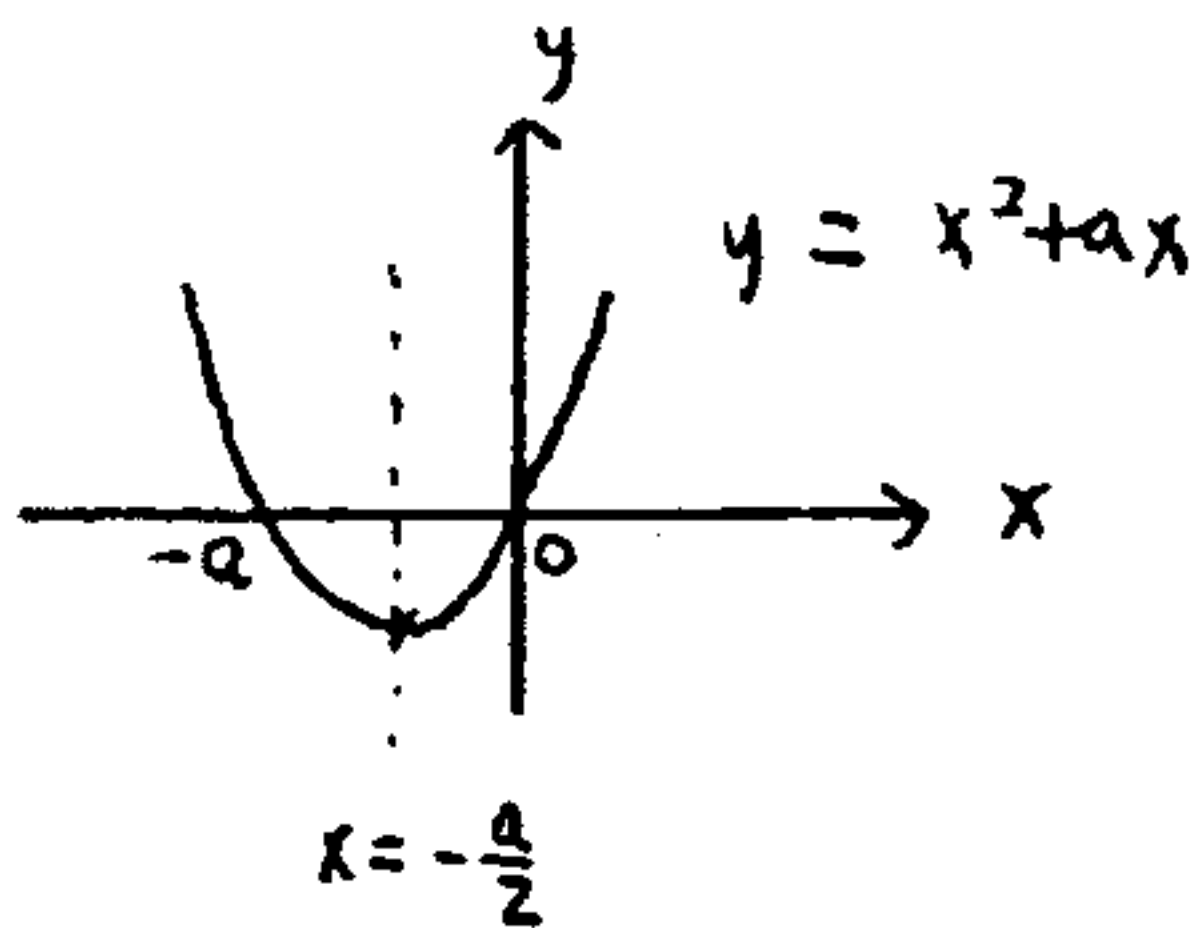
Section A

1)



Hence $|x+1| \leq e^{1-x}$
 $\Rightarrow -1.57 \leq x \leq 0.557$

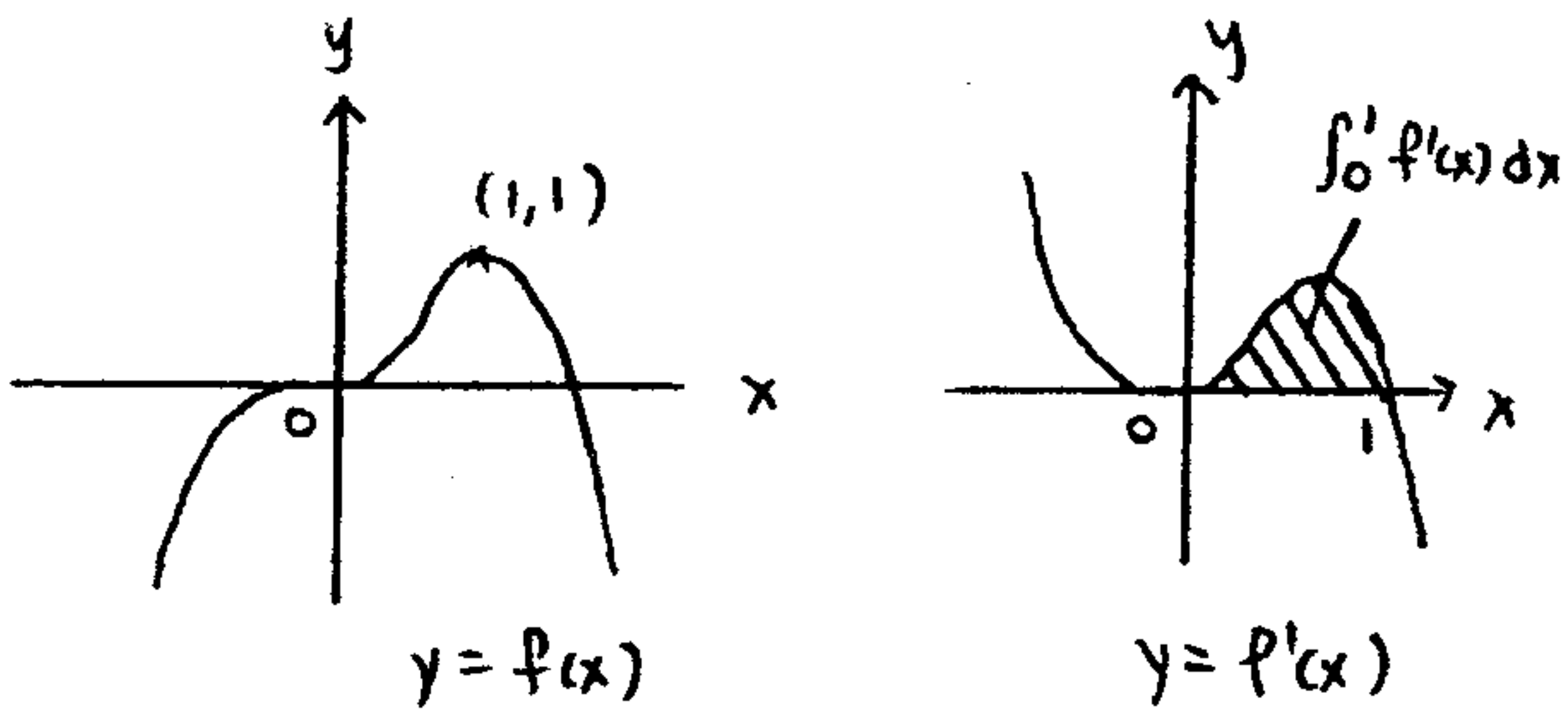
2) (i)



Since $x \in [b, \infty)$, for f^{-1} to exist,
 least value of $b = -\frac{a}{2}$

(ii) $(3, 1)$ is a pt on $y = f^{-1}(x)$
 $\Rightarrow 1 = f^{-1}(3)$
 $\Rightarrow f(1) = 3$
 $\therefore 1+a = 3$
 $a = 2$

3)



$$\int_0^1 f'(x) dx = [f(x)]_0^1$$

$$= f(1) - f(0)$$

$$= 1 - 0$$

$$= 1$$

4) (i)

$$\frac{dA}{dt} = \frac{3}{3t+1}$$

Since $t \geq 0$, $\frac{dA}{dt} > 0$
 So A is \uparrow with time

$$\frac{dV}{dt} = 0.2 e^{0.2t} > 0 \quad (\because e^{0.2t} > 0)$$

So V is \uparrow with time

(ii) $\frac{dA}{dt} = 0.1875 \Rightarrow \frac{3}{3t+1} = 0.1875$
 $\Rightarrow t = 5$

$\therefore \frac{dV}{dt} = 0.2 e^{0.2(5)}$
 $= 0.544 \text{ (3sf)}$

5) $y = \frac{2-x}{x^2}$

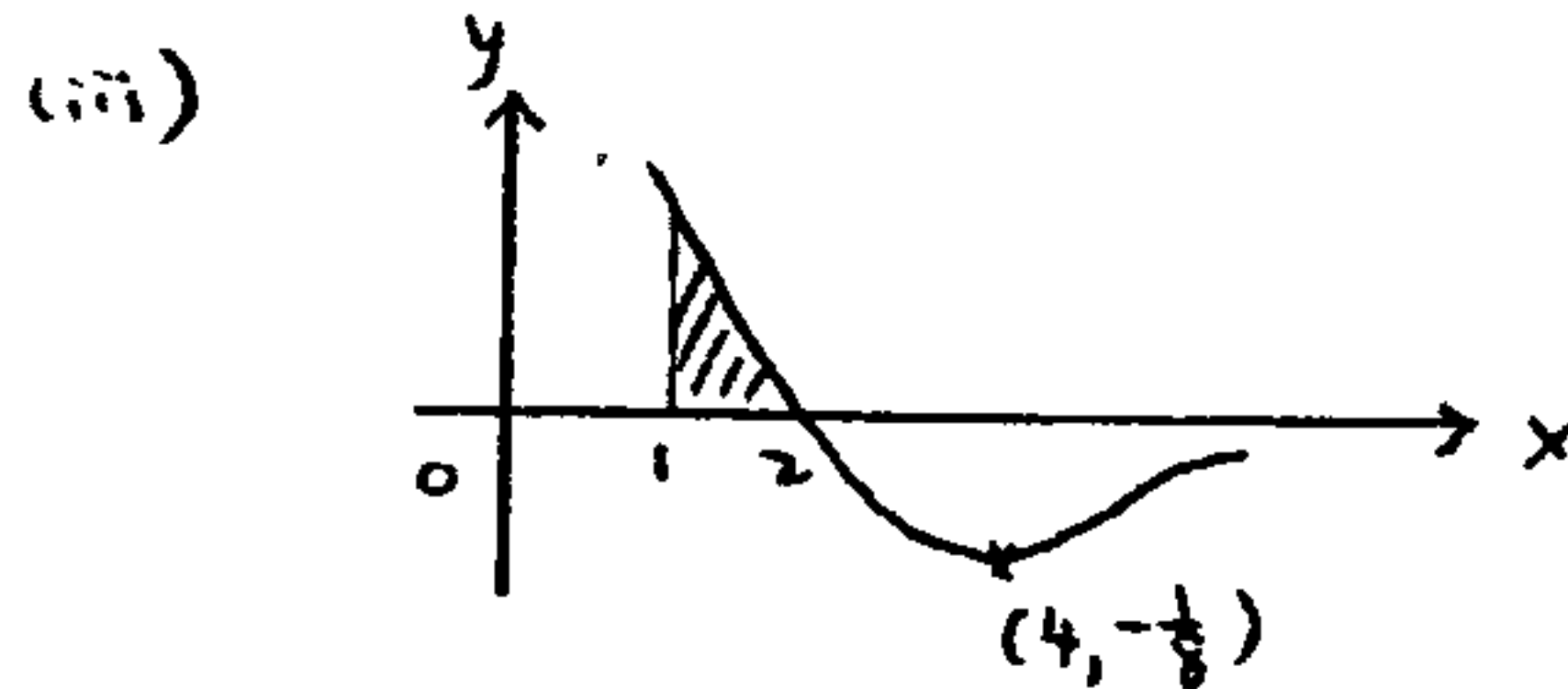
(i) $y = 2x^{-2} - x^{-1}$
 $\frac{dy}{dx} = -4x^{-3} + x^{-2}$
 For minimum pt, $\frac{dy}{dx} = 0$
 $\Rightarrow \frac{4}{x^3} = \frac{1}{x^2}$
 $x = 4$

When $x = 4$, $y = \frac{2-4}{4^2} = -\frac{1}{8}$

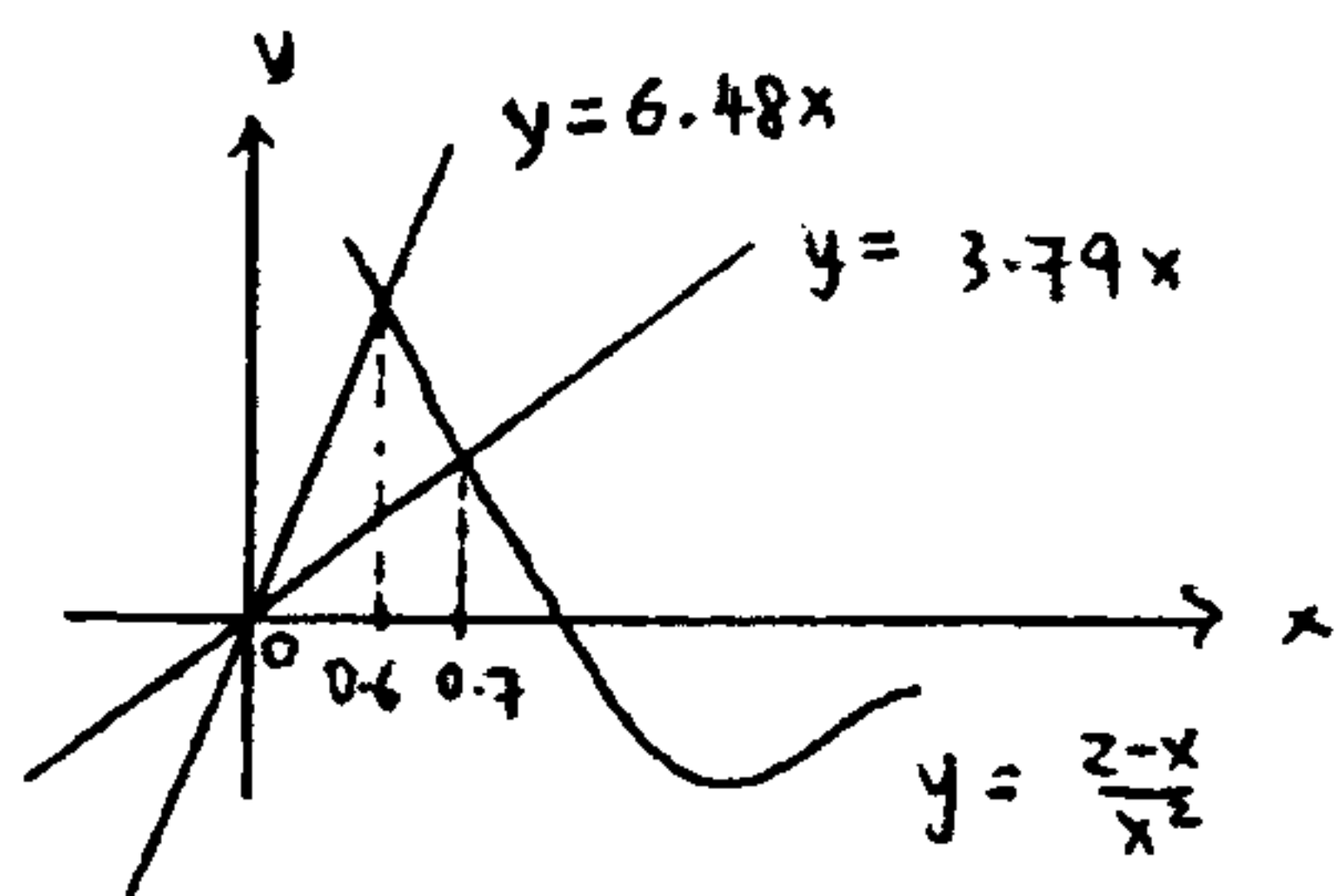
\therefore Coordinates of minimum pt $(4, -\frac{1}{8})$
 sign of $\frac{dy}{dx}$ $\frac{-}{+} \frac{+}{-}$

(ii) At $x = 3$, $\frac{dy}{dx} = -\frac{1}{27}$, $y = -\frac{1}{9}$

\therefore Eqn of normal is
 $y + \frac{1}{9} = 27(x-3)$
 $9y = 243x - 730$



Reqd area = $\int_1^2 \frac{2-x}{x^2} dx$
 $= \int_1^2 (2x^{-2} - x^{-1}) dx$
 $= [-2x^{-1} - \ln|x|]_1^2$
 $= (-1 - \ln 2) - (-2)$
 $= 1 - \ln 2$



When $x = 0.6$, $y = 6.48x$ ($m = 6.48$)
 $x = 0.7$, $y = 3.79x$ ($m = 3.79$)

$\therefore m = 4, 5, 6$
 since $3.79 < m < 6.48$

Section B

6) $X \sim B(n, p)$
 $E(X) = 12 \Rightarrow np = 12 \quad \text{--- (1)}$
 $\text{Var}(X) = 4 \Rightarrow np(1-p) = 4 \quad \text{--- (2)}$
 $(2) \div (1): \quad 1-p = \frac{1}{3}$
 $p = \frac{2}{3}$
 $\therefore n(\frac{2}{3}) = 12$
 $n = 18$

7) Let X be the mass of a student
 Let T be the total mass of 50 students
 $\therefore T = X_1 + X_2 + \dots + X_{50}$
 By CLT, $T \sim N(50 \times 51, 50 \times 121)$
 $T \sim N(2550, 6050) \text{ approx}$
 $P(T > 2600) = 0.260 \text{ (3sf)}$

8) (i) Quota Sampling
 (ii) No. This is because each resident in the small town is not equally likely to be chosen for the interview.
 (iii) Use stratified sampling

C	34	} 40
M	3	
I	2	
E	1	

Pick 34 Chinese, 3 Malays, 2 Indians & 1 Eurasian randomly.

9) $\sum(x-200) = -25 \Rightarrow \bar{x} = 200 + \frac{(-25)}{50}$
 $= 199.5$
 $s^2 = \frac{1}{49} [159.5 - \frac{(-25)^2}{50}]$
 $= 3$

$H_0: \mu = 200$
 $H_1: \mu < 200$
 Test Statistics: $Z = \frac{\bar{X} - 200}{\frac{s}{\sqrt{n}}}$

From GC, $p = 0.0206 < 0.03$
 $\therefore H_0$ is rejected at 3% significance level
 There is significant evidence at the 3% significance level that the population mean mass of the content is less than 200g.

$H_0: \mu = 200$
 $H_1: \mu \neq 200$
 If H_1 is accepted, $2p < \alpha\%$
 $\therefore \alpha > 4.12$

10) $A \sim N(115, 26^2) \quad \bar{A} = \frac{1}{3}(A_1 + A_2 + A_3) \sim N(115, \frac{26^2}{3})$
 $B \sim N(104, 21^2) \quad \bar{B} = \frac{1}{4}(B_1 + \dots + B_4) \sim N(104, \frac{21^2}{4})$
 $\therefore \bar{A} - \bar{B} \sim N(11, 335.58)$
 $P(\bar{A} > \bar{B}) = P(\bar{A} - \bar{B} > 0) = 0.726$

11) 10 R 5 G 3 B
 (i) Prob = $\frac{10}{18} \cdot \frac{9}{17} + \frac{5}{18} \cdot \frac{4}{17} + \frac{3}{18} \cdot \frac{2}{17}$
 $= \frac{58}{153}$
 (ii) Prob = $\frac{8}{17}$ (9 R 8 non R)
 (iii) $P(1^{st} R | 2^{nd} R) = \frac{P(1^{st} R \& 2^{nd} R)}{P(2^{nd} R)}$
 $= \frac{\frac{10}{18} \cdot \frac{9}{17}}{\frac{10}{18} \cdot \frac{9}{17} + \frac{8}{18} \cdot \frac{10}{17}}$
 $= \frac{9}{17}$

12) $X \sim N(\mu, \sigma^2)$
 $P(X > 10) = 0.179 \Rightarrow P(Z > \frac{10-\mu}{\sigma}) = 0.179$
 $\Rightarrow \frac{10-\mu}{\sigma} = 0.91918 \text{ --- (1)}$
 $P(X < 9) = 0.720 \Rightarrow P(Z < \frac{9-\mu}{\sigma}) = 0.720$
 $\Rightarrow \frac{9-\mu}{\sigma} = 0.58284 \text{ --- (2)}$

Solving (1) & (2), $\mu = 7.27, \sigma^2 = 8.84$

Let Y denote the no of observations (out of 6) that will be more than 10.

$Y \sim B(6, 0.179)$

$\therefore P(Y \geq 2) = 1 - P(Y \leq 1) = 0.293$

Let W denote the no of observations (out of 60) that will be less than 9.

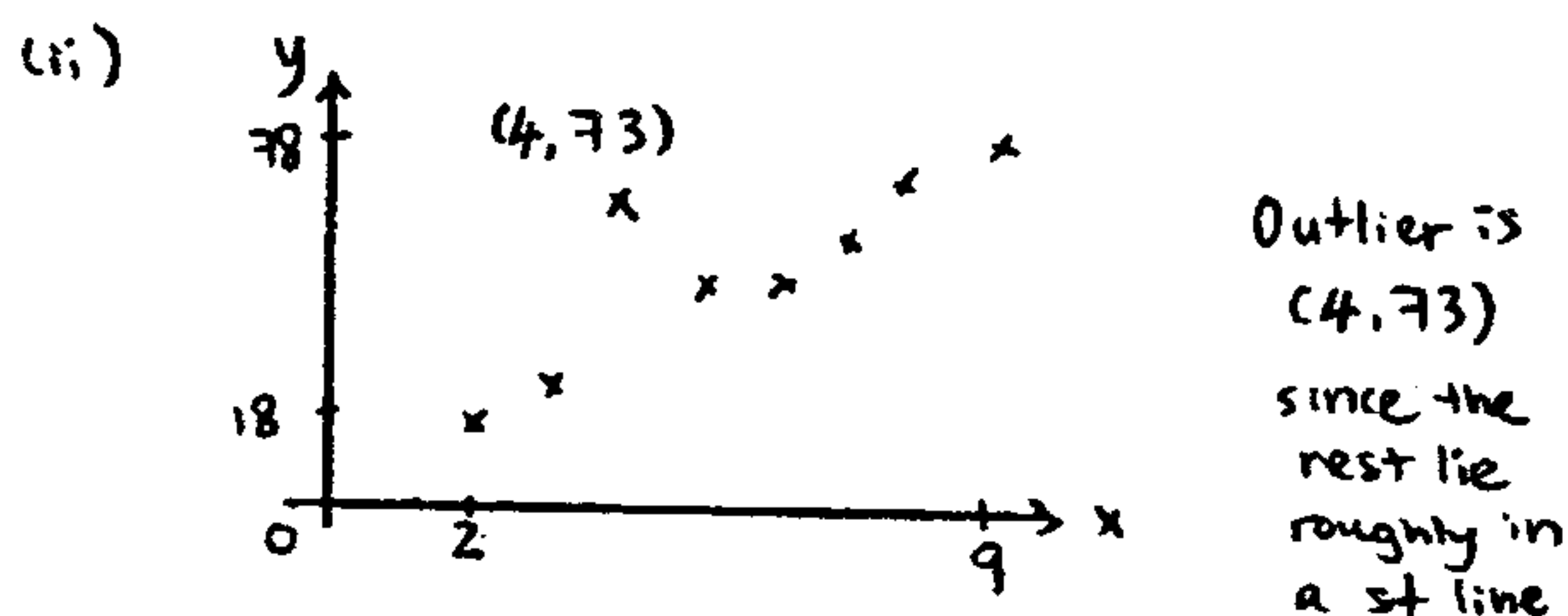
$W \sim B(60, 0.720)$

Since $np = 43.2 > 5, n(1-p) = 16.8 > 5$

$W \sim N(43.2, 12.096) \text{ approx}$

$P(W \geq 44) \stackrel{cc}{=} P(W \geq 43.5) = 0.466$

13) (i) From GC, $r = 0.813$



(iii) From GC, new $r = 0.995$

Since $r \approx 1$, there is a positive linear correlation between no of weeks and mark obtained

(iv) $y = 0.30435 + 8.7717x$

(a) $x=4, y=35.4$ reliable

(b) $x=12, y=105.6$ not reliable since outside the range